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Delay-dependent Stability Criteria for a Class of Uncertain Neutral Systems

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Abstract In this paper, the problem of delay-dependent stability criteria for a class of constant time-delay neutral systems with time-varying structured uncertainties is investigated. New asymptotic stability criteria are derived in the form of linear matrix inequality. Two numerical examples are given to indicate that the results presented in this paper are effective and less conservative than some existing results.

Key words Neutral systems, delay-dependent criteria, time-varying structured uncertainties, asymptotically stable, linear matrix inequality

It is well known that time-delay is often the important source of instability, which is encountered in various engineering systems. So the stability analysis for time-delay systems has great importance both in theory and in practice.

In the past years, many researchers have paid attention to the stability analysis of time-delay neutral systems and achieved many results[1−13]. The existing results on this topic can be classified into two categories, namely, delay-dependent and delay-independent. Generally speaking, the latter is more conservative than the former.

In [3−4], some delay-independent stability criteria are given in terms of the characteristic equation of system, involving the measures, eigenvalues, spectral radius and spectral norms of the corresponding matrices. Although these conditions are easy to check, they require the matrix measure to be negative or the matrix to be Hurwitz matrix. Using Lyapunov method, [5] derived a new delay-independent stability condition by linear matrix inequality, which is easily solved by convex optimization algorithms. But [3−5] all ignored the information of the time-delay, which caused conservatism of the criteria especially when the value of time-delay is comparatively small. Recently, [5, 9−10, 12] presented delay-dependent criteria in the form of linear matrix inequality. On the basis of [5, 9−10, 12], [6−8] took the relationship between the terms in the Leibniz-Newton formula into account. They also chose some free-weight matrix and gave some new stability criteria, which improved some previous results. But there is room for furthermore investigation.

In this paper, we will extend the method derived by Liu[8] to the constant time-delay neutral system with time-varying constructed uncertainties. A new delay-dependent stability criterion will be given, which can reduce the conservatism of some previous results.

We consider the information contained in the nominal neutral system equation

$$\dot{x}(t) - C\dot{x}(t-h) = Ax(t) + Bx(t-h)$$

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From the system equation, we can know that

$$\int_{t-h}^{t} Ax(s) + Bx(s-h)\mathrm{d}s = \int_{t-h}^{t} \dot{x}(s) - C\dot{x}(s-h)\mathrm{d}s = x(t) - (C+I)x(t-h) + Cx(t-2h)$$

and we can choose some suitable free-weight matrices to express the relationships among vectors $x(t)$, $x(t-h)$, and $x(t-2h)$. Based on this information and Leibniz-Newton formula, a new stability criterion for such a nominal neutral system is derived, which is formulated in the form of a linear matrix inequality. Moreover, it can be easily extended to the systems with time-varying structured uncertainties. Finally, two numerical examples are given to illustrate that the criteria given in this paper augment the allowed maximum upper bounds in previous results and are less conservative.

For simplicity, through the paper, the following notations are used: $\mathbf{R}^n$ denotes the n-dimensional real space; $\mathbf{R}^{n\times n}$ denotes the $n\times n$-dimensional real matrix; A^{T} denotes the transpose of the matrix A; $\|A\|$ denotes the Euclidean norm of the matrix A, namely, $\|A\| = \lambda_{\max}^{1/2}(A^{\mathrm{T}}A)$; $P>0$ is a positive-definite symmetric matrix; $P\geq 0$ is a positive semi-definite symmetric matrix; I denotes the identity matrix with appropriate dimensions, and $*$ denotes the symmetric part in the matrix.

1 Problem formulation

Consider the following uncertain neutral system Σ

$$\begin{cases} \dot{x}(t) - C\dot{x}(t-h) = (A+\Delta A(t))x(t) + \\ \qquad (B+\Delta B(t))x(t-h), & t>0 \\ x(t) = \phi(t), & t\in[-h,0] \end{cases} \tag{1}$$

where $x(t)\in\mathbf{R}^n$ is the state vector; $A, B, C\in\mathbf{R}^{n\times n}$ are constant matrices; and h is the constant time-delay. The initial condition $\phi(t)$ is a continuous differentiable function on $[-h,0]$. The time-varying structured uncertainties $\Delta A(t)$ and $\Delta B(t)$ are of the form

$$[\Delta A(t), \Delta B(t)] = DF(t)[E_1, E_2] \tag{2}$$

where D, E_1, and E_2 are appropriate dimensional constant matrices. $F(t)$ is an unknown time-varying real matrix with Lebesgue measurable elements satisfying

$$F(t)^{\mathrm{T}}F(t)\leq I, \quad \forall t\geq 0 \tag{3}$$

First, we consider the nominal system Σ_0 of Σ

$$\begin{cases} \dot{x}(t) - C\dot{x}(t-h) = Ax(t) + Bx(t-h), & t>0 \\ x(t) = \phi(t), & t\in[-h,0] \end{cases} \tag{4}$$

To guarantee that the difference operator $\Gamma: C[-h,0]\to\mathbf{R}^n$ given by

$\Gamma(x_t) = x(t) - Cx(t-h)$ is stable, we assume[1]

$$\|C\| < 1 \tag{5}$$

where $\|\cdot\|$ is any matrix norm.

To obtain the results for the system with time-varying structured uncertainties, we employ the following lemma.

Lemma 1[2]. Given matrices $Q=Q^{\mathrm{T}}, G, E$, and $R=R^{\mathrm{T}}>0$ of appropriate dimensions, then $Q+GFE+E^{\mathrm{T}}F^{\mathrm{T}}G^{\mathrm{T}}<0$ for all F satisfying $F^{\mathrm{T}}F\leq R$ if and only if there exists some $\lambda>0$, such that $Q+\lambda^{-1}GG^{\mathrm{T}}+\lambda E^{\mathrm{T}}RE<0$.