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ON DECENTRALIZED STABILIZATION OF LINEAR LARGE SCALE SYSTEMS WITH SYMMETRIC CIRCULANT STRUCTURE *

JIN Chao-yong (金朝永)[1], ZHANG Xiang-wei (张湘伟)[2]

(1. College of Applied Mathematics, Guangdong University of Technology, Guangzhou 510090, P.R.China;
2. College of Electromechanical Engineering, Guangdong University of Technology, Guangzhou 510090, P.R.China)

(Contributed by ZHANG Xiang-wei)

Abstract: *The decentralized stabilization of continuous and discrete linear large scale systems with symmetric circulant structure was studied. A few sufficient conditions on decentralized stabilization of such systems were proposed. For the continuous systems, by introducing a concept called the magnitude of interconnected structure, a very important property that the decentralized stabilization of such systems is fully determined by the structure of each isolated subsystem that is obtained when the magnitude of interconnected structure of the overall system is given. So the decentralized stabilization of such systems can be got by only appropriately designing or modifying the structure of each isolated subsystem, no matter how complicated the interconnected structure of the overall system is. A algorithm for obtaining decentralized state feedback to stabilize the overall system is given. The discrete systems were also discussed. The results show that there is a great dfference on decentralized stabilization between continuous case and discrete case.*

Key words: large scale system; decentralized stabilization; symmetric circulant structure; magnitude of interconnected structure; Ricatti equation

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Introduction

The problem of stabilizing the overall linear large scale system by employing a decentralized state feedback control for each subsystems has received much attention in the past two decades because it is very important in theory and application. Generally speaking, in spite of the

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Biographies: JIN Chao-yong (1962 ~), Associate Professor (E-mail: cyjin8@hotmail.com); ZHANG Xiang-wei (Corresponding author, Tel: +86-20-37628870; Fax: +86-20-37626119; E-mail: xwzhang@gdut.edu.cn)

controllable and observable hypothesis, it is not always possible to find the decentralized control law with the desired stabilizing property. Therefore it is necessary to make some additional conditions about the interconnection matrix, and many results were given[1~6].

In this paper, we study a class of linear large scale systems with symmetric circulant structure, which have been studied in several papers[7~9]. Such systems are very common in practice including paper machines, distribution networks, and systems consisting of units operating in parallel. By using the concept called the magnitude of interconnected structure, we establish some sufficient conditions to assure the decentralized stabilization of such systems. For the continuous systems, the results obtained show that the decentralized stabilization of such systems is fully determined by the structure (A, B) of each isolated subsystem when the magnitude of interconnected structure of the overall system is given, no matter how complicated the interconnected structure is. We also discuss the discrete systems, the results show that there is a great difference on decentralized stabilization between continuous case and discrete case.

This paper is organized as follows. In Section 1, the model of large scale systems with symmetric circulant structure is given. Sections 2 and 3 are devoted to discussing the properties of decentralized stabilization of such systems for continuous case and discrete case, respectively. Two illustrative examples are given in Section 4.

1 Problem Formulation

First we give the definition of block symmetric circulant matrix.

Definition 1.1 A matrix $\boldsymbol{H} \in R^{Nm\times Np}$ is called block circulant if $\boldsymbol{H}$ has the following structure:

$$\boldsymbol{H} = \begin{pmatrix} \boldsymbol{H}_1 & \boldsymbol{H}_2 & \cdots & \boldsymbol{H}_N \\ \boldsymbol{H}_N & \boldsymbol{H}_1 & \cdots & \boldsymbol{H}_{N-1} \\ \cdots & \cdots & \cdots & \cdots \\ \boldsymbol{H}_2 & \boldsymbol{H}_3 & \cdots & \boldsymbol{H}_1 \end{pmatrix}, \tag{1}$$

where $\boldsymbol{H}_i \in R^{m\times p}$ $(i = 1, 2, 3, \cdots, N)$. Moreover, if $\boldsymbol{H}_i = \boldsymbol{H}_{N-i+2}$ $(i = 2, 3, \cdots, N)$, then the matrix $\boldsymbol{H}$ is called block symmetric circulant, which is denoted by $\mathrm{scl}[\boldsymbol{H}_1, \boldsymbol{H}_2, \cdots, \boldsymbol{H}_N]$.

Let $\boldsymbol{m}_j = (1, v_j, v_j^2, \cdots, v_j^{N-1})^{\mathrm{T}}$, $j = 1, 2, \cdots, N$, where $v_j = \exp[(2\pi(j-1)\sqrt{-1})/N]$, $(j = 1, 2, 3, \cdots, N)$, i.e., v_j is the jth root of the equation $v^N = 1$. Let $\boldsymbol{R}_N = \frac{1}{\sqrt{N}}[\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots, \boldsymbol{r}_N]$ with $\boldsymbol{r}_1 = \boldsymbol{m}_1 = [1, 1, \cdots, 1]^{\mathrm{T}}$, $\boldsymbol{r}_{\frac{N}{2}+1} = \boldsymbol{m}_{\frac{N}{2}+1}$ if N is an even number,

$\boldsymbol{r}_p = \frac{1}{\sqrt{2}}(\boldsymbol{m}_p + \boldsymbol{m}_{N+2-p})$, $\boldsymbol{r}_{N+2-p} = \frac{\sqrt{-1}}{\sqrt{2}}(\boldsymbol{m}_p - \boldsymbol{m}_{N+2-p})$, $p = 2, 3, \cdots, l$, where $l = \frac{N+1}{2}$ if N is odd and $l = \frac{N}{2}$ if N is even.

Lemma 1.1[8,9] The matrix $\boldsymbol{R}_N$ is a real orthogonal matrix, and satisfies

$$\boldsymbol{E}_k^{-1} = \boldsymbol{E}_k^{\mathrm{T}}, \tag{2}$$

where $\boldsymbol{E}_k = \boldsymbol{R}_N \otimes \boldsymbol{I}_k$, $\otimes$ denotes the Kronecker product, and $\boldsymbol{I}_k$ denotes the $k\times k$ identity matrix, $\boldsymbol{E}_k^{\mathrm{T}}$ denotes the transposition of $\boldsymbol{E}_k$.

Lemma 1.2[8,9] Let $\boldsymbol{H} = \mathrm{scl}[\boldsymbol{H}_1, \boldsymbol{H}_2, \cdots, \boldsymbol{H}_N]$ with $\boldsymbol{H}_i \in R^{m\times p}$ $(i = 1, 2, \cdots, N)$, and

$\boldsymbol{H}_d = \boldsymbol{E}_m^{\mathrm{T}}\boldsymbol{H}\boldsymbol{E}_p$. Then $\boldsymbol{H}_d =$ block diag$\left\{\boldsymbol{H}_{d_1}, \boldsymbol{H}_{d_2}, \cdots, \boldsymbol{H}_{d_N}\right\}$ is a block diagonal matrix with $\boldsymbol{H}_{d_i} = \boldsymbol{H}_{d_{N+2-i}}$ ($i = 2, 3, \cdots, l$). Moreover, $\boldsymbol{H}_i$ and $\boldsymbol{H}_{d_i}$ satisfy

$$\begin{pmatrix} \boldsymbol{H}_{d_1} \\ \boldsymbol{H}_{d_2} \\ \vdots \\ \boldsymbol{H}_{d_N} \end{pmatrix} = (\sqrt{N}\boldsymbol{F}_N \otimes \boldsymbol{I}_m)^{\mathrm{T}} \begin{pmatrix} \boldsymbol{H}_1 \\ \boldsymbol{H}_2 \\ \vdots \\ \boldsymbol{H}_N \end{pmatrix}, \quad \begin{pmatrix} \boldsymbol{H}_1 \\ \boldsymbol{H}_2 \\ \vdots \\ \boldsymbol{H}_N \end{pmatrix} = \frac{1}{\sqrt{N}}\boldsymbol{F}_N(\otimes \boldsymbol{I}_m) \begin{pmatrix} \boldsymbol{H}_{d_1} \\ \boldsymbol{H}_{d_2} \\ \vdots \\ \boldsymbol{H}_{d_N} \end{pmatrix}, \tag{3}$$

where $\boldsymbol{F}_N^{\mathrm{T}} = \frac{1}{\sqrt{N}}[\boldsymbol{m}_1\ \boldsymbol{m}_2 \cdots \boldsymbol{m}_N]$.

Let Σ be a dynamic linear time-invariant system defined by N interconnected subsystems Σ_i, $i = 1, 2, \cdots N$. For the continuous case

$$\Sigma_i: \begin{cases} \dot{\boldsymbol{x}}_i(t) = \boldsymbol{A}\boldsymbol{x}_i(t) + \boldsymbol{B}\boldsymbol{u}_i(t) + \sum\limits_{j=1, j\neq i}^{N} \boldsymbol{D}_{ij}\boldsymbol{x}_j(t), \\ \boldsymbol{y}_i(t) = \boldsymbol{C}\boldsymbol{x}_i(t) \qquad (i = 1, 2, \cdots, N), \end{cases} \tag{4}$$

and for the discrete case

$$\Sigma_i: \begin{cases} \boldsymbol{x}_i(k+1) = \boldsymbol{A}\boldsymbol{x}_i(k) + \boldsymbol{B}\boldsymbol{u}_i(k) + \sum\limits_{j=1, j\neq i}^{N} \boldsymbol{D}_{ij}\boldsymbol{x}_j(k), \\ \boldsymbol{y}_i(k) = \boldsymbol{C}\boldsymbol{x}_i(k) \qquad (i = 1, 2, \cdots, N), \end{cases} \tag{5}$$

where in both models, $\boldsymbol{A} \in R^{n\times n}$, $\boldsymbol{B} \in R^{n\times m}$, $\boldsymbol{C} \in R^{r\times n}$, $\boldsymbol{D}_{ij} \in R^{n\times n}$ ($i \neq j = 1, 2, \cdots, N$) are constant matrices and $\boldsymbol{x}_i \in R^n$, $\boldsymbol{u}_i \in R^m$, $\boldsymbol{y}_i \in R^r$ represent the state, input and output of the subsystem Σ_i, respectively.

Let $\boldsymbol{x} = (\boldsymbol{x}_1^{\mathrm{T}}, \boldsymbol{x}_2^{\mathrm{T}}, \cdots, \boldsymbol{x}_N^{\mathrm{T}})^{\mathrm{T}}$, $\boldsymbol{y} = (\boldsymbol{y}_1^{\mathrm{T}}, \boldsymbol{y}_2^{\mathrm{T}}, \cdots, \boldsymbol{y}_N^{\mathrm{T}})^{\mathrm{T}}$, $\boldsymbol{u} = (\boldsymbol{u}_1^{\mathrm{T}}, \boldsymbol{u}_2^{\mathrm{T}}, \cdots, \boldsymbol{u}_N^{\mathrm{T}})^{\mathrm{T}}$, $\bar{\boldsymbol{A}} =$ block diag$(\boldsymbol{A}, \boldsymbol{A}, \cdots, \boldsymbol{A})$, $\boldsymbol{B} =$ block diag$(\boldsymbol{B}, \boldsymbol{B}, \cdots, \boldsymbol{B})$, $\bar{\boldsymbol{C}} =$ block diag$(\boldsymbol{C}, \boldsymbol{C}, \cdots, \boldsymbol{C})$, and $\boldsymbol{D} = (\boldsymbol{D}_{ij}) \in R^{Nn\times Nn}$ with $\boldsymbol{D}_{ii} = \boldsymbol{0}$ ($i = 1, 2, \cdots, N$). Employ a decentralized state feedback control for each subsystems

$$\boldsymbol{u}_i(t) = \boldsymbol{K}_i\boldsymbol{x}_i(t) \qquad \text{(for the continuous case)}(i = 1, 2, \cdots, N), \tag{6}$$

$$\boldsymbol{u}_i(k) = \boldsymbol{K}_i\boldsymbol{x}_i(k) \qquad \text{(for the discrete case)}(i = 1, 2, \cdots, N), \tag{7}$$

where $\boldsymbol{K}_i \in R^{m\times n}$ is the matrix of decentralized gain, $i = 1, 2, \cdots, N$.

From Eqs. (4) ~ (7) we can write the closed-loop overall system as

$$\begin{cases} \dot{\boldsymbol{x}}(t) = (\bar{\boldsymbol{A}} + \boldsymbol{B}\boldsymbol{K} + \boldsymbol{D})\boldsymbol{x}(t), \\ \boldsymbol{y}(t) = \bar{\boldsymbol{C}}\boldsymbol{x}(t) \end{cases} \tag{8}$$

for the continuous case and

$$\begin{cases} \boldsymbol{x}(k+1) = (\bar{\boldsymbol{A}} + \boldsymbol{B}\boldsymbol{K} + \boldsymbol{D})\boldsymbol{x}(k), \\ \boldsymbol{y}(k) = \bar{\boldsymbol{C}}\boldsymbol{x}(k) \end{cases} \tag{9}$$

for the discrete case. Where in both Eq. (8) and Eq. (9), $\boldsymbol{K} =$ block diag $(\boldsymbol{K}_1, \boldsymbol{K}_2, \cdots, \boldsymbol{K}_N)$.

Definition 1.2 The system (4) (or the system (5)) is called a continuous (or discrete) linear large scale system with symmetric circulant structure if the interconnection matrix $\boldsymbol{D} = (\boldsymbol{D}_{ij})$ is block symmetric circulant, i. e., $\boldsymbol{D} = \mathrm{scl}[\boldsymbol{0}, \boldsymbol{D}_2, \cdots, \boldsymbol{D}_N]$.

The system described in definition 1.2 has two properties: one is that all the isolated subsystems have the same structure as $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})$, the other is that the interconnection matrix $\boldsymbol{D}$ is block symmetric circulant.

Definition 1.3 The continuous system (4) (or discrete system (5)) is called decentralized stabilization if there exists a decentralized state feedback control (6) (or (7)) for each subsystems Σ_i such that the closed-loop overall system (8) (or (9)) is asymptotically stable.

Our general goal is to reveal some fundamental properties of the decentralized stabilization about continuous and discrete linear large scale systems with symmetric circulant structure. In following discussion we always assume that the interconnection matrix $\boldsymbol{D}=\mathrm{scl}[\boldsymbol{0}, \boldsymbol{D}_2, \cdots, \boldsymbol{D}_N]$, $(\boldsymbol{A}, \boldsymbol{B})$ is completely controllable and $(\boldsymbol{A}, \boldsymbol{C})$ is completely observable, it follows that for all $\beta>0$, the matrix Ricatti equations

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{PA}-\boldsymbol{PBB}^{\mathrm{T}}\boldsymbol{P}+\beta\boldsymbol{I}_n=\boldsymbol{0} \quad \text{(for the continuous case)} \tag{10}$$

and

$$\boldsymbol{P}=\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PA}-\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PB}(\boldsymbol{I}_m+\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PB})^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PA}+\beta\boldsymbol{I}_n \quad \text{(for the discrete case)} \tag{11}$$

exist a unique solution $\boldsymbol{P}$ respectively which is symmetric and positive definite (denoted by $\boldsymbol{P}>\boldsymbol{0}$).

Let $\overline{\boldsymbol{P}}=$ block diag$(\boldsymbol{P}, \boldsymbol{P}, \cdots, \boldsymbol{P})$, then from Eqs.(10) and (11) we have

$$\overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{P}}+\overline{\boldsymbol{P}}\overline{\boldsymbol{A}}-\overline{\boldsymbol{P}}\overline{\boldsymbol{B}}\overline{\boldsymbol{B}}^{\mathrm{T}}\overline{\boldsymbol{P}}+\beta\boldsymbol{I}_{Nn}=\boldsymbol{0} \quad \text{(for the continuous case)} \tag{12}$$

and

$$\overline{\boldsymbol{P}}=\overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{P}}\overline{\boldsymbol{A}}-\overline{\boldsymbol{A}}^{\mathrm{T}}\overline{\boldsymbol{P}}\overline{\boldsymbol{B}}(\boldsymbol{I}_{Nm}+\overline{\boldsymbol{B}}^{\mathrm{T}}\overline{\boldsymbol{P}}\overline{\boldsymbol{B}})^{-1}\overline{\boldsymbol{B}}^{\mathrm{T}}\overline{\boldsymbol{P}}\overline{\boldsymbol{A}}+\beta\boldsymbol{I}_{Nn} \quad \text{(for the discrete case).} \tag{13}$$

Furthermore, from Lemma 1.1 and Lemma 1.2, we can easily get the following results:

$$\begin{cases} \boldsymbol{A}_d \overset{\text{def}}{=} \boldsymbol{E}_n^{-1}\overline{\boldsymbol{A}}\boldsymbol{E}_n=\overline{\boldsymbol{A}}=\text{block diag}(\boldsymbol{A}, \boldsymbol{A}, \cdots, \boldsymbol{A}), \\ \boldsymbol{B}_d \overset{\text{def}}{=} \boldsymbol{E}_n^{-1}\overline{\boldsymbol{B}}\boldsymbol{E}_m=\overline{\boldsymbol{B}}=\text{block diag}(\boldsymbol{B}, \boldsymbol{B}, \cdots, \boldsymbol{B}), \\ \boldsymbol{C}_d \overset{\text{def}}{=} \boldsymbol{E}_r^{-1}\overline{\boldsymbol{C}}\boldsymbol{E}_n=\overline{\boldsymbol{C}}=\text{block diag}(\boldsymbol{C}, \boldsymbol{C}, \cdots, \boldsymbol{C}), \\ \boldsymbol{D}_d \overset{\text{def}}{=} \boldsymbol{E}_n^{-1}\boldsymbol{D}\boldsymbol{E}_n=\text{block diag}(\boldsymbol{D}_{d_1}, \boldsymbol{D}_{d_2}, \cdots, \boldsymbol{D}_{d_N}), \\ \boldsymbol{P}_d \overset{\text{def}}{=} \boldsymbol{E}_n^{-1}\overline{\boldsymbol{P}}\boldsymbol{E}_n=\overline{\boldsymbol{P}}=\text{block diag}(\boldsymbol{P}, \boldsymbol{P}, \cdots, \boldsymbol{P}), \end{cases} \tag{14}$$

where $\boldsymbol{D}_{d_i}=\boldsymbol{D}_{d_{N+2-i}}$ $(i=2, 3, \cdots, l)$.

Moreover, we denote $\lambda_{\max}(\boldsymbol{P})$ and $\lambda_{\min}(\boldsymbol{P})$ as the maximum and minimum eigenvalue of the symmetric matrix $\boldsymbol{P}$, respectively. Let $\|\circ\|$ denote the 2-norm, i.e., $\|\boldsymbol{A}\|=(\lambda_{\max}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}))^{\frac{1}{2}}$.

Definition 1.4 Let $\rho(\boldsymbol{D})=\lambda_{\max}(\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D})$. We call $\rho(\boldsymbol{D})$ the magnitude of interconnected structure for both systems (4) and (5).

Lemma 1.3 $\rho(\boldsymbol{D})=\|\boldsymbol{D}_d\|^2=\max\limits_{1\leqslant i\leqslant N}\|\boldsymbol{D}_{d_i}\|^2$.

Proof

$$\begin{aligned}\max_{1\leqslant i\leqslant N}\|\boldsymbol{D}_{d_i}\|^2 &= \max_{1\leqslant i\leqslant N}\lambda_{\max}(\boldsymbol{D}_{d_i}^{\mathrm{T}}\boldsymbol{D}_{d_i})=\lambda_{\max}(\boldsymbol{D}_d^{\mathrm{T}}\boldsymbol{D}_d)= \\ &\lambda_{\max}(\boldsymbol{E}_n^{-1}\boldsymbol{D}^{\mathrm{T}}\boldsymbol{E}_n\boldsymbol{E}_n^{-1}\boldsymbol{D}\boldsymbol{E}_n)=\lambda_{\max}(\boldsymbol{E}_n^{-1}\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D}\boldsymbol{E}_n)= \\ &\lambda_{\max}(\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D})=\rho(\boldsymbol{D}).\end{aligned}$$

The proof is completed.

2 The Properties of Decentralized Stabilization for the Continuous systems

We first consider the continuous linear system (8). Our main result is stated in the following theorem.

Theorem 2.1 Let β be a positive scalar, P be the unique symmetric and positive definite solution of the continuous Ricatti equation (10). If

$$\beta > \|\boldsymbol{P}\|^2 + \rho(\boldsymbol{D}), \tag{15}$$

then the decentralized gain matrices

$$\boldsymbol{u}_i(t) = -\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}_i(t) \qquad (i = 1, 2, \cdots, N) \tag{16}$$

guarantee the asymptotic stability of the closed-loop overall system (8).

Proof Substitution of Eq. (16) into Eq. (8) yields the closed-loop overall system

$$\dot{\boldsymbol{x}}(t) = (\overline{\boldsymbol{A}} - \overline{\boldsymbol{B}}\overline{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{D})\boldsymbol{x}(t). \tag{17}$$

We choose $v(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}$ as a valid Liapunov function for the system (17). Taking its time derivative along the solution of Eq. (17) and by using Eqs. (12) and (14), we have

$$\begin{aligned}
\dot{v}(x) = {} & \boldsymbol{x}^{\mathrm{T}}(\overline{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\overline{\boldsymbol{A}} - 2\boldsymbol{P}\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{D})\boldsymbol{x} = \\
& \boldsymbol{x}^{\mathrm{T}}(-\beta\boldsymbol{I}_{Nn} - \overline{\boldsymbol{P}}\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{D}^{\mathrm{T}}\overline{\boldsymbol{P}} + \overline{\boldsymbol{P}}\boldsymbol{D})\boldsymbol{x} \leqslant \\
& \boldsymbol{x}^{\mathrm{T}}(-\beta\boldsymbol{I}_{Nn} + \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{D})\boldsymbol{x} = \\
& \boldsymbol{x}^{\mathrm{T}}\boldsymbol{E}_n(-\beta\boldsymbol{I}_{Nn} + \boldsymbol{E}_n^{-1}\boldsymbol{D}^{\mathrm{T}}\boldsymbol{E}_n\boldsymbol{E}_n^{-1}\overline{\boldsymbol{P}}\boldsymbol{E}_n + \boldsymbol{E}_n^{-1}\overline{\boldsymbol{P}}\boldsymbol{E}_n\boldsymbol{E}_n^{-1}\boldsymbol{D}\boldsymbol{E}_n)\boldsymbol{E}_n^{-1}\boldsymbol{x} = \\
& -\boldsymbol{z}^{\mathrm{T}}(\beta\boldsymbol{I}_{Nn} - \boldsymbol{D}_d^{\mathrm{T}}\boldsymbol{P}_d - \boldsymbol{P}_d\boldsymbol{D}_d)\boldsymbol{z} = \\
& -\boldsymbol{z}^{\mathrm{T}}(\beta\boldsymbol{I}_{Nn} + (\boldsymbol{P}_d - \boldsymbol{D}_d)^{\mathrm{T}}(\boldsymbol{P}_d - \boldsymbol{D}_d) - \boldsymbol{P}_d^2 - \boldsymbol{D}_d^{\mathrm{T}}\boldsymbol{D}_d)\boldsymbol{z} \leqslant \\
& -\boldsymbol{z}^{\mathrm{T}}(\beta\boldsymbol{I}_{Nn} - \boldsymbol{P}_d^2 - \boldsymbol{D}_d^{\mathrm{T}}\boldsymbol{D}_d)\boldsymbol{z},
\end{aligned}$$

where $\boldsymbol{z} = \boldsymbol{E}_n^{-1}\boldsymbol{x}$. It is obvious that $\boldsymbol{x} \neq \boldsymbol{0}$ if and only if $\boldsymbol{z} \neq \boldsymbol{0}$. Now condition (15) together with Eq. (14) and Lemma 1.3 implies $(\beta\boldsymbol{I}_{Nn} - \boldsymbol{P}_d^2 - \boldsymbol{D}_d^{\mathrm{T}}\boldsymbol{D}_d) > \boldsymbol{0}$. Hence we have $\dot{v}(x) < 0$ for $\boldsymbol{x} \neq \boldsymbol{0}$ and $\dot{v}(\boldsymbol{x}) = 0$, which assures the asymptotic stability of the closed-loop overall system (17) by employing the decentralized gain matrices $K_i = -B^TP$, $i = 1, 2, \cdots, N$. The proof is completed.

In order to test the condition (15), we should calculate $\rho(\boldsymbol{D})$, which may involve a very difficult numerical calculation. In practice, if we can formerly choose a positive scalar $\rho > 0$ such that $\rho(\boldsymbol{D}) \leqslant \rho$, and take ρ instead of $\rho(\boldsymbol{D})$ in the condition (15), the result of Theorem 2.1 also holds.

Corollary 2.1 Let $\rho > 0$ be a positive scalar such that $\rho(\boldsymbol{D}) \leqslant \rho$, $\boldsymbol{P}$ be the unique symmetric and positive definite solution of the continuous Ricatti equation (10). If

$$\beta > \|\boldsymbol{P}\|^2 + \rho, \tag{18}$$

then the decentralized gain matrices (16) guarantee the asymptotic stability of the closed-loop overall system (8).

Remarks We are now in the position to make several observations concerning the results established so far.

1) The magnitude $\rho(\boldsymbol{D})$ or the upper bound ρ of $\rho(\boldsymbol{D})$ can be determined beforehand according to the interconnected structure of the overall system. The Ricatti equation (10) shows that the relationship between β and $\boldsymbol{P}$ is fully determined by the structure $(\boldsymbol{A}, \boldsymbol{B})$ of each isolated subsystem. Those mean in practice we can get the property of the decentralized stabilization for the linear systems (4) by only appropriately designing or modifying $(\boldsymbol{A}, \boldsymbol{B})$ to satisfy the condition (15) or (18), no matter how complicated the overall interconnected structure $\boldsymbol{D}$ is. It is impossible to do that for the general interconnected systems.

2) Because in Eq. (16) the decentralized gain matrices $\boldsymbol{K}_i$ $(i = 1, 2, \cdots, N)$ for all isolated subsystems are the same, in practice we can first design or modify $(\boldsymbol{A}, \boldsymbol{B})$ to satisfy the condition (15) or (18) for a special isolated system, then copy it to the rest. We must note that from the proof of Theorem 2.1 we see that the same structure $(\boldsymbol{A}, \boldsymbol{B})$ for every isolated subsystems is necessary.

3) To choose β satisfying the condition (15) or (18) is difficult because in the Ricatti equation (10) the relationship between the scalar β and the matrix $\boldsymbol{P}$ is nonlinear. Generally speaking, the larger β is, the easier to satisfy condition (15) or (18) but the higher decentralized gain will be. So under condition (15) or (18) we should choose the positive scalar β as small as possible in practice.

Based on the foregoing relations, the stabilization of the continuous linear large scale systems with symmetric circulant structure by means of decentralized state feedback control law (6) can be performed by the following algorithm:

Step 1 Calculate $\rho(\boldsymbol{D})$ or estimate an upper bound ρ for the $\rho(\boldsymbol{D})$;

Step 2 Give β a positive initial value, substitute it into Eq.(10) and solve for $\boldsymbol{P}$;

Step 3 Verify whether the condition (15) or (18) holds. If yes, then turn to Step 6, and otherwise, to Step 4;

Step 4 Enlarge β in the way of Remark 3), and turn to Step 2;

Step 5 If the condition (15) or (18) does not hold for sufficiently large β (for example $\beta \geqslant M$, where M is a bound given formerly), we appropriately modify some parameters of $(\boldsymbol{A}, \boldsymbol{B})$ in the way of Remarks 1) and 2), then turn to Step 2;

Step 6 Take the decentralized state feedback control law in the following:

$$\boldsymbol{u}_i(t) = -\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}_i(t) \qquad (i = 1, 2, \cdots, N).$$

In Theorem 2.1, the existence of the positive parameter β satisfying the condition (15) or (18) is essential for obtaining the decentralized gains (16). However, in general, it is not always possible to find the decentralized gains $\boldsymbol{K}_i (i = 1, 2, \cdots, N)$ which will stabilize the overall system (8).

Let us now consider a typical case where all the interconnection matrices $\boldsymbol{D}_i (i = 2, 3, \cdots, N)$ can be factored as

$$\begin{cases} \boldsymbol{D}_i = \boldsymbol{B}\boldsymbol{L}_i\boldsymbol{C} \\ \boldsymbol{L}_i = \boldsymbol{L}_{N+2-i} \in R^{m\times r} \end{cases} \qquad (i = 2, 3, \cdots, N) \tag{19}$$

which is the special case to Theorem 2.1. Let $\boldsymbol{L} = \mathrm{scl}[\boldsymbol{0}, \boldsymbol{L}_2, \boldsymbol{L}_3, \cdots, \boldsymbol{L}_N]$, then we have

$$\boldsymbol{L}_d \overset{\mathrm{def}}{=} \boldsymbol{E}_m^{-1}\boldsymbol{L}\boldsymbol{E}_r = \text{block diag}(\boldsymbol{L}_{d_1}, \boldsymbol{L}_{d_2}, \cdots, \boldsymbol{L}_{d_N}), \tag{20}$$

where $\boldsymbol{L}_{d_i} = \boldsymbol{L}_{d_{N+2-i}} (i = 2, 3, \cdots, l)$.

Theorem 2.2 Under the hypothesis (19), the overall continuous system (8) is stabilizable by means of a local state feedback control (6), where the decentralized gain matrix $\boldsymbol{K}_i$ is determined by

$$\begin{cases} \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P} + \beta\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C} = \boldsymbol{0}, \\ \boldsymbol{K}_i = -\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P} \qquad (i = 1, 2, \cdots, N). \end{cases} \tag{21}$$

Here β is a positive scalar satisfies

$$\beta > \max_{1\leqslant i\leqslant N} \| \boldsymbol{L}_{d_i} \|^2. \tag{22}$$

Proof Due to Eq.(19) we have $\boldsymbol{D} = \boldsymbol{B}\boldsymbol{L}\bar{\boldsymbol{C}}$. Following the similar argument in Theorem 2.1, we can choose $v(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}$ as a valid Liapunov function for the system (17). Taking its time derivative along the solution of the system (17) and by using Eq. (22) we obtain

$$\begin{aligned} \dot{v}(x) = {} & \boldsymbol{x}^{\mathrm{T}}(-\beta\bar{\boldsymbol{C}}^{\mathrm{T}}\bar{\boldsymbol{C}} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P} + \bar{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{L}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{B}\boldsymbol{L}\bar{\boldsymbol{C}})\boldsymbol{x} = \\ & \boldsymbol{x}^{\mathrm{T}}(-\beta\bar{\boldsymbol{C}}^{\mathrm{T}}\bar{\boldsymbol{C}} - (\boldsymbol{L}\bar{\boldsymbol{C}} - \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P})^{\mathrm{T}}(\boldsymbol{L}\bar{\boldsymbol{C}} - \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}) + \bar{\boldsymbol{C}}^{\mathrm{T}}\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}\bar{\boldsymbol{C}})\boldsymbol{x}. \end{aligned}$$

Define $\boldsymbol{G} = \boldsymbol{L}\bar{\boldsymbol{C}} - \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}$, $\boldsymbol{z}(t) = \boldsymbol{E}_r^{-1}\boldsymbol{y}(t)$, where $\boldsymbol{y}(t) = \bar{\boldsymbol{C}}\boldsymbol{x}(t)$, then

$$\begin{aligned}\dot{v}(x) = &-\boldsymbol{y}^{\mathrm{T}}(\beta\boldsymbol{I}_{Nr} - \boldsymbol{L}^{\mathrm{T}}\boldsymbol{L})\boldsymbol{y} - \boldsymbol{x}^{\mathrm{T}}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}\boldsymbol{x} = \\ &-\boldsymbol{y}^{\mathrm{T}}\boldsymbol{E}_r(\beta\boldsymbol{I}_{Nr} - \boldsymbol{E}_r^{-1}\boldsymbol{L}^{\mathrm{T}}\boldsymbol{E}_m\boldsymbol{E}_m^{-1}\boldsymbol{L}\boldsymbol{E}_r)\boldsymbol{E}_r^{-1}\boldsymbol{y} - \boldsymbol{x}^{\mathrm{T}}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}\boldsymbol{x} = \\ &-\boldsymbol{z}^{\mathrm{T}}(\beta\boldsymbol{I}_{Nr} - \boldsymbol{L}_d^{\mathrm{T}}\boldsymbol{L}_d)\boldsymbol{z} - \boldsymbol{x}^{\mathrm{T}}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}\boldsymbol{x} \leqslant \\ &-\boldsymbol{z}^{\mathrm{T}}(\beta\boldsymbol{I}_{Nr} - \boldsymbol{L}_d^{\mathrm{T}}\boldsymbol{L}_d)\boldsymbol{z}.\end{aligned} \tag{23}$$

Now due to Eq. (22), $\beta\boldsymbol{I}_{Nr} - \boldsymbol{L}_d^{\mathrm{T}}\boldsymbol{L}_d > \boldsymbol{0}$, so we have $\dot{v}(x) \leqslant 0$, which proves the stability of the overall continuous system (17). To prove the asymptotic stability, it is sufficient to remember that the solution of Eq. (17) is $\boldsymbol{x}(t) = \exp\{(\bar{\boldsymbol{A}} - \boldsymbol{B}\boldsymbol{G})t\}\boldsymbol{x}_0$ for arbitrary $\boldsymbol{x}(0) = \boldsymbol{x}_0 \neq \boldsymbol{0}$, it follows that a $\boldsymbol{x}_0 \neq \boldsymbol{0}$ such that $\dot{v}(\boldsymbol{x}(t)) \equiv 0$, $\forall t \geqslant 0$ cannot exist because in this case otherwise, by using Eq. (23), we would have

$$\begin{cases}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}\boldsymbol{x} = 0 \\ \boldsymbol{z}^{\mathrm{T}}(\beta\boldsymbol{I}_{Nr} - \boldsymbol{L}_d^{\mathrm{T}}\boldsymbol{L}_d)\boldsymbol{z} = 0\end{cases} \quad (\forall t \geqslant 0), \tag{24}$$

which implies that

$$\bar{C}\exp(\bar{A}t)x_0 = 0 \quad (\forall t \geqslant 0).$$

However, this contradicts the observability of the pair $(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}})$. Hence, the proof is completed.

3 The Properties of Decentralized Stabilization for the Discrete Systems

Now we consider the discrete linear system (5) or (9). Our purpose in this section is to generalize the previous results to the discrete case. Using similar argument as in the continuous case, we can get the following main results.

Theorem 3.1 Let β be a positive scalar, $\boldsymbol{P}$ be the unique symmetric and positive definite solution of the discrete Ricatti equation (11). If

$$\beta\boldsymbol{I}_n - \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A} - \boldsymbol{D}_{d_i}^{\mathrm{T}}(2\boldsymbol{P} + \boldsymbol{P}\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P})\boldsymbol{D}_{d_i} > \boldsymbol{0} \quad (i = 1, 2, \cdots, N), \tag{25}$$

then decentralized state feedback

$$\boldsymbol{u}_i(k) = -(\boldsymbol{I}_m + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B})^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A}\boldsymbol{x}_i(k) \quad (i = 1, 2, \cdots, N) \tag{26}$$

guarantee the asymptotic stability of the closed-loop overall system (9).

If all the interconnection matrices $\boldsymbol{D}_i$ $(i = 2, 3, \cdots, N)$ in the discrete system (5) can be factored as Eq. (19), we have

Theorem 3.2 Given the hypothesis (19), if there exists a positive scalar β such that the unique symmetric and positive definite solution $\boldsymbol{P}$ of the discrete Ricatti equation (11) satisfies

$$\beta\boldsymbol{I}_r - \boldsymbol{L}_{d_i}^{\mathrm{T}}(\boldsymbol{I}_m + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B})\boldsymbol{L}_{d_i} > \boldsymbol{0} \quad (i = 1, 2, \cdots, N), \tag{27}$$

then the overall discrete system (5) is stabilizable by means of a local state feedback control (26).

Now we can summarize the properties of decentralized stabilization for the discrete linear large scale systems with symmetric circulant structure according to the continuous case.

1) Because the condition (25) is related to the overall interconnected matrix $\boldsymbol{D}$ (not $\rho(\boldsymbol{D})$), generally we can't get the decentralized stabilization only by designing or modifying the isolated structure $(\boldsymbol{A}, \boldsymbol{B})$ even the overall interconnected structure magnitude $\rho(\boldsymbol{D})$ is given, which is very different from the continuous case.

2) The system with symmetric circulant structure described by condition (19) can always be stabilized by a decentralized state feedback control (6) for the continuous case, but it does not

hold for the discrete case.

3) Remarks 1) and 2) show that there exists a great difference on the properties of decentralized stabilization between continuous case and discrete case. Generally speaking, the latter is more complicated than the former. We must notice it in practice.

4 Example Analysis

Example 1 Consider a class of continuous linear large scale systems with symmetric circulant structure

$$\begin{cases} \dot{\boldsymbol{x}}_i(t) = -\delta \boldsymbol{x}_i(t) + \boldsymbol{B}\boldsymbol{u}_i(t) + \sum_{j=1, j\neq i}^{N} \boldsymbol{D}_{ij}\boldsymbol{x}_j(t), \\ \boldsymbol{y}_i(t) = \boldsymbol{C}\boldsymbol{x}_i(t) \qquad (i = 1, 2, \cdots, N), \end{cases} \tag{28}$$

where $\delta > 0$, $\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}} = \boldsymbol{I}_n$, $\boldsymbol{D} = (\boldsymbol{D}_{ij})$ is block symmetric circulant. In this case the continuous Ricatti equation (10) can be simplified as

$$\boldsymbol{P}^2 + 2\delta\boldsymbol{P} - \beta\boldsymbol{I}_n = \boldsymbol{0}. \tag{29}$$

The unique symmetric and positive definite solution $\boldsymbol{P}$ of Eq. (29) is $\boldsymbol{P} = (\sqrt{\delta^2 + \beta} - \delta)\boldsymbol{I}_n$. By computing directly, we obtain that the condition (15) will hold when β satisfies

$$\beta > \left(1 + \frac{\rho(\boldsymbol{D})}{4\delta^2}\right)\rho(\boldsymbol{D}). \tag{30}$$

For all given $\delta > 0$ and block symmetric circulant interconnected matrix $\boldsymbol{D}$, we can always choose a sufficient positive scalar β satisfying the condition (30). Hence, the overall continuous system (28) is always stabilizable by means of a local state feedback control law given by

$$\boldsymbol{u}_i(t) = -(\sqrt{\delta^2 + \beta} - \delta)\boldsymbol{B}^{\mathrm{T}}\boldsymbol{x}_i(t) \qquad (i = 1, 2, \cdots, N),$$

where positive scalar β satisfies the condition (30).

Example 2 Consider a continuous linear system with symmetric circulant structure, where the overall system consists of N linearly interconnected 2-dimension subsystems, described by

$$\begin{cases} \dot{\boldsymbol{x}}_i(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \boldsymbol{x}_i(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \boldsymbol{u}_i(t) + \sum_{j=1, j\neq i}^{N} \boldsymbol{D}_{ij}\boldsymbol{x}_j(t), \\ \boldsymbol{y}_i(t) = \boldsymbol{C}\boldsymbol{x}_i(t) \qquad (i = 1, 2, \cdots, N). \end{cases} \tag{31}$$

By computing directly, $\forall \beta > 0$, the unique symmetric and positive definite solution $\boldsymbol{P}$ of the Ricatti equation

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\boldsymbol{P} + \boldsymbol{P}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \boldsymbol{P}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \end{bmatrix}\boldsymbol{P} + \beta\boldsymbol{I}_2 = \boldsymbol{0}$$

is

$$\boldsymbol{P} = \begin{bmatrix} \sqrt{\beta^2 + 2\beta\sqrt{\beta}} & \sqrt{\beta} \\ \sqrt{\beta} & \sqrt{\beta + 2\sqrt{\beta}} \end{bmatrix}.$$

It's easy to test that $\beta < \|\boldsymbol{P}\|^2$, so for system (31), the condition (15) in Theorem 2.1 cannot hold for arbitrary positive scalar $\beta > 0$. In order to assure the decentralized stabilization of the system (31), we slightly modify the structure $(\boldsymbol{A}, \boldsymbol{B})$ into $(\bar{\boldsymbol{A}}, \bar{\boldsymbol{B}})$ for each subsystems, where

$$\bar{\boldsymbol{A}} = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}, \quad \bar{\boldsymbol{B}} = \begin{bmatrix} 0 \\ \sqrt{\alpha} \end{bmatrix},$$

where $\alpha > 0$ is the positive scalar to be determined. Taking $\beta = \alpha$, then we have

$$\begin{bmatrix} 0 & 0 \\ \alpha & 0 \end{bmatrix} \boldsymbol{P} + \boldsymbol{P} \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix} - \boldsymbol{P} \begin{bmatrix} 0 \\ \sqrt{\alpha} \end{bmatrix} \begin{bmatrix} 0 & \sqrt{\alpha} \end{bmatrix} \boldsymbol{P} + \alpha \boldsymbol{I}_2 = \boldsymbol{0}.$$

The unique symmetric and positive definite solution $\boldsymbol{P}$ of the last Ricatti equation is

$$\boldsymbol{P} = \begin{bmatrix} \sqrt{3} & 1 \\ 1 & \sqrt{3} \end{bmatrix}.$$

By computing directly, we have $\| \boldsymbol{P} \|^2 = 7.464\,1$, hence according to Theorem 2. 1, the modified system is decentralized stabilization for arbitrary interconnected matrix $\boldsymbol{D}$ if we take $\beta = \alpha > 7.464\,1 + \rho(\boldsymbol{D})$.

It must be noted that the way of choosing parameters α and β is not unique. For example, we can choose them in the following: first let $\beta = \gamma\alpha$, then for each given γ, the Ricatti equation

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \boldsymbol{P} + \boldsymbol{P} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \boldsymbol{P} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \boldsymbol{P} + \gamma \boldsymbol{I}_2 = \boldsymbol{0}$$

provides us a unique symmetric and positive definite solution $\boldsymbol{P}_\gamma$. Finally we take $\beta > \| \boldsymbol{P}_\gamma \|^2 + \rho(\boldsymbol{D})$, $\alpha = \beta / \gamma$. The following table gives the relationship among α, β and γ for $\rho(\boldsymbol{D}) = 1$.

Table 1

γ	$\| \boldsymbol{P} \|^2$	β	α
0.25	1.991 8	2.991 8	11.967 2
0.5	3.673 0	4.673 0	9.346
1	7.464 1	8.464 1	8.464 1
2	17.124 7	18.124 7	9.062 4
4	44.784 2	45.784 2	11.446 1

From the above table, we can see that the larger the γ, the larger the β and the $\| \boldsymbol{P} \|$. On the other hand, the larger the γ, the smaller the α. Hence in practice, we can obtain a group of suitable values of β, $\| \boldsymbol{P} \|$ and α by adjusting γ.

5 Conclusion

We have studied the decentralized stabilization of the continuous and discrete linear large scale systems with symmetric circulant structure. For the continuous case the results obtained in the paper provide us a practical way of decentralized stabilization by only appropriately designing or modifying the structure $(\boldsymbol{A}, \boldsymbol{B})$ of each isolated subsystems, no matter how complicated the overall interconnected structure $\boldsymbol{D}$ is. Unfortunately, it does not suit the discrete case. Therefore it is necessary to develop the theory of decentralized stabilization further for the discrete linear interconnected systems.

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